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Supersymmetry breaking and α'-corrections to flux induced potentials

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Abstract: We obtain the vacuum solutions for M-theory compactified on eight-manifolds with non-vanishing four-form flux by analyzing the scalar potential appearing in the three-dimensional theory. Many of these vacua are not supersymmetric and yet have a vanishing three-dimensional cosmological constant.We show that in the context of type-IIB compactifications on Calabi-Yau threefolds with fluxes and external brane sources α'-corrections generate a correction to the supergravity potential proportional to the Euler number of the internal manifold which spoils the no-scale structure appearing in the classical potential. This indicates that α'-corrections may indeed lead to a stabilization of the radial modulus appearing in these compactifications.

Keywords: M-Theory, Superstring Vacua, Supersymmetry Breaking.

Contents

1. Introduction

Compactifications of M-theory on a Calabi-Yau fourfold result in a theory with $\mathcal{N} = 2$ supersymmetry in three dimensions which is roughly equivalent to $\mathcal{N} = 1$ supersymmetry in four dimensions. However, there is no evidence for supersymmetry in the observed low energy spectrum, so that physics below the compactification scale must break supersymmetry in such a way that the cosmological constant remains extremely small. Some time ago gaugino condensation was suggested as a possible mechanism for hierarchical supersymmetry breaking in compactifications of the heterotic string [1, 2]. Augmented with the possibility of a background flux for the field strength of the antisymmetric tensor [3] it was observed that in simple models a stable vacuum with broken supersymmetry and vanishing cosmological constant can be arranged. However, it has been problematic to implement a large hierarchy of scales and a realistic pattern of soft supersymmetry breaking terms [4]. Recently a similar scenario has been revived in the context of warped type-IIB compactifications on Calabi-Yau threefolds with three-form fluxes and localized sources [5, 6, 7]. The potential induced by the fluxes is of the no-scale type. This implies that a non-vanishing (0,3)-flux breaks supersymmetry [8, 9] without introducing a vacuum energy.[1] Supersymmetry is generically broken at a scale that depends on the volume of the Calabi-Yau threefold. In [16] an analogous version of this mechanism was found for compactifications of M-theory on eight-manifolds with non-vanishing fluxes for tensor fields.[2] The constraints imposed by supersymmetry on such compactifications were determined in [17]. It was shown in [18] that these constraints can be derived from two superpotentials that describe the vacuum solutions in three dimensions. These superpotentials and the corresponding scalar potential were obtained by a Kaluza-Klein reduction of M-theory on a fourfold in [19, 20].[3]

[1] For the closely related $\mathcal{N} = 2$ vacua the potential and low energy effective action were derived in [10]–[14] and the vacuum structure was investigated in [15].

[2] The models found in [5] can be derived from [16] if the eight-manifold is an elliptic fibration.

[3] A similar Kaluza-Klein reduction of type-IIA theory on Calabi-Yau fourfolds with background fluxes is performed in [21, 22] while the potential for M-theory on G_2-holonomy manifolds with fluxes has been computed in [23].

In this note we extend the analysis of [5, 16] in two directions. First we rederive the results of [16] by determining the minima of the potential calculated in [20]. This analysis will also show that the non-supersymmetric vacua found in [16] are classically stable. Our second aim is to investigate the fate of the no-scale structure of the potential if one considers the effect of higher derivative corrections of string theory and M-theory. We confirm the expectation of [5] that the no-scale structure in type-IIB compactifications does not survive in the quantum theory once higher order α'-corrections are taken into account. In particular, this implies that breaking supersymmetry via a (0,3)-flux induces a non-vanishing potential. Due to the relationship between type-IIB compactifications with three-form flux and M-theory compactifications with four-form flux [18, 24] a similar result should be valid for the non-supersymmetric fluxes in M-theory [16], which also lead to a vanishing cosmological constant at leading order.

This paper is organized as follows. In section 2 we rederive the results of [16] from the scalar potential derived in [20] and show that the non-supersymmetric vacua are classically stable. In section 3 we calculate higher order α'^3-corrections to the scalar potential computed in [5]. We show that these corrections generate a scalar potential that depends on the Calabi-Yau volume and is proportional to the Euler number of the internal manifold. This spoils the no-scale structure of the classical scalar potential for manifolds with non-vanishing Euler number and suggests that further α'-corrections may lead to a stabilization of the radial modulus. Some of the technical details of the computation are relegated to appendix A.

2. (Non)-supersymmetric solutions in M-theory

In this section we derive the non-supersymmetric vacuum solutions with vanishing cosmological constant computed in [16] from the superpotentials found in [18, 20]. We use the notation and conventions of [20].

2.1 The scalar potential

The scalar potential of M-theory compactified on a fourfold Y_4 to three dimensions takes the form[4]

$$V = e^{K^{(3)}} G^{-1\alpha\bar{\beta}} D_\alpha W D_{\bar{\beta}} \bar{W} + \left(\frac{1}{2} G^{-1AB} \partial_A \hat{W} \partial_B \hat{W} - \hat{W}^2 \right), \tag{2.1}$$

where $\alpha, \beta = 1, \ldots, h^{3,1}$ label the complex structure deformations while $A, B = 1, \ldots, h^{1,1}$ label the deformations of the Kähler structure. The Kähler potential[5] $K^{(3)}$ is given in terms of the holomorphic four-form Ω and the fourfold volume $\mathcal{V}$

$$K^{(3)} = -\ln \int_{Y_4} \Omega \wedge \bar{\Omega} + \ln \mathcal{V}, \tag{2.2}$$

[4]We express the formulas in this section in terms of the rescaled Kähler coordinates $\hat{M}^A = \mathcal{V}^{-1} M^A$ used in [20]. However, in order not to overload the notation we omit the hat on the coordinates here.

[5]Strictly speaking $K^{(3)}$ is not a Kähler potential since in three dimensions the Kähler deformations M^A are not complexified. Nevertheless, the metric G_{AB} is determined by the second derivative of $K^{(3)}$.

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while

$$W = \int_{Y_4} F \wedge \Omega \,, \tag{2.3}$$

and

$$\hat{W} = \frac{1}{4}\int_{Y_4} F \wedge J \wedge J \tag{2.4}$$

represent the two superpotentials depending on the four-form flux $F.G_{AB} = -\frac{1}{2}\partial_A\partial_B K^{(3)}$ and $G_{\alpha\bar{\beta}} = \partial_\alpha\partial_{\bar{\beta}}K^{(3)}$ are the metrics on the moduli spaces of Kähler- and complex structure deformations, respectively, and we further defined $D_\alpha W = \partial_\alpha W + (\partial_\alpha K^{(3)})W$.

The scalar potential (2.1) originates entirely from the anti-selfdual internal components of the four-form F [20] as the selfdual part cancels out from tadpole cancellation. To see this, expand the four-form flux as $F = F_{4,0} + F_{3,1} + F_{2,2} + F_{1,3} + F_{0,4}$ and furthermore use the Lefschetz decomposition in order to further expand $F_{2,2}$ as[6]

$$F_{2,2} = F^{(0)}_{2,2} + J \wedge F^{(0)}_{1,1} + J^2 \wedge F^{(0)}_{0,0} \,. \tag{2.5}$$

Here $F^{(0)}_{p,p}$ denotes a primitive (p,p)-form on Y_4, i.e. it satisfies

$$J^{5-2p} \wedge F^{(0)}_{p,p} = 0 \,. \tag{2.6}$$

It has been shown in [18, 20] that $F_{3,1}$, $F_{1,3}$ and $J \wedge F^{(0)}_{1,1}$ are anti-selfdual, whereas the other four-forms are selfdual. Furthermore, the potential (2.1) can alternatively be rewritten as [20]

$$V = -\mathcal{V}\left(\int_{Y_4} F_{3,1} \wedge F_{1,3} + \frac{1}{2}\int_{Y_4} J \wedge F^{(0)}_{1,1} \wedge J \wedge F^{(0)}_{1,1}\right) . \tag{2.7}$$

Thus the potential only depends on the anti-selfdual part of the four-form F. It was noticed in [18] that this scalar potential is of no-scale type since it is independent of the volume (up to an overall multiplicative factor coming from the Weyl-rescaling) and hence also does not depend on the radial modulus of the fourfold. The first term in the brackets of (2.7) is manifestly volume independent. To see this also for the second term, we rescale J by a factor λ. This amounts to rescaling the volume by λ^4 and requires also to rescale $F^{(0)}_{1,1}$ with λ^{-1} (and $F^{(0)}_{0,0}$ with λ^{-2}) in order to keep $F_{2,2}$ in (2.5) unchanged. Thus a rescaling of the volume precisely drops out of the combination $F^{(0)}_{1,1} \wedge J$ and implies that also the second term in (2.7) does not depend on the radial modulus. Furthermore, V is non-negative which can be seen by rewriting (2.1) once more as

$$V = e^{K^{(3)}} G^{-1\alpha\bar{\beta}} D_\alpha W D_{\bar{\beta}}\bar{W} + \frac{1}{2} G^{-1AB} D_A\hat{W} D_B\hat{W} \,, \tag{2.8}$$

where we introduced the covariant derivative $D_A\hat{W} = \partial_A\hat{W} - \frac{1}{2}\left(\partial_A K^{(3)}\right)\hat{W}$. Written in this form V is manifestly non negative.

[6]Note that this decomposition slightly differs from the one in [20] in that we use the rescaled Kähler form here.

2.2 Supersymmetry

In supergravity theories with four supercharges the conditions for unbroken supersymmetry for compactifications to three-dimensional Minkowski space are [18]

$$W = D_\alpha W = 0\,, \tag{2.9}$$

and

$$\hat{W} = \partial_A \hat{W} = 0\,. \tag{2.10}$$

The constraints for a supersymmetric three-dimensional vacuum found in [17] then easily follow from the above conditions [18, 20]. First, $W = 0$ implies $F_{4,0} = F_{0,4} = 0$. Second, the identity $D_\alpha W = \int \Phi_\alpha \wedge F$, where Φ_α is a basis of $(3,1)$-forms, gives together with $D_\alpha W = 0$ the condition $F_{1,3} = F_{3,1} = 0$. Third, the condition $\partial_A \hat{W} = 0$ implies that F is primitive

$$F \wedge J = 0\,, \tag{2.11}$$

while $\hat{W} = 0$ follows from the above condition and imposes no new constraint.

Let us now consider the non-supersymmetric solutions. Taking the leading quantum gravity corrections of M-theory into account it was shown in [16] that turning on a flux of the form

$$F = F_{4,0} + F_{0,4} + F_{0,0} J \wedge J\,, \tag{2.12}$$

breaks supersymmetry without generating a cosmological constant. This can also easily be seen from the previous discussion. From the potential (2.8) we see that the solution to the equations of motion has to satisfy

$$D_\alpha W = D_A \hat{W} = 0\,. \tag{2.13}$$

The vanishing of $D_\alpha W$ implies $F_{1,3} = F_{3,1} = 0$, while $D_A \hat{W} = 0$ implies $F^{(0)}_{1,1} \wedge J = 0$. Therefore we see that the flux has to be selfdual. It is clear from (2.8) that theses solutions do not generate a cosmological constant. A flux of the form $F = F_{4,0} + F_{0,4}$ breaks supersymmetry as $W \neq 0$, while a flux $F \sim J \wedge J$ is not primitive implying $\hat{W} \neq 0$ and supersymmetry is broken in this case as well. These non-supersymmetric vacua are stable to this order as the potential (2.1) is non negative. The solution $V = 0$ then corresponds to a minimum of the potential and a vanishing of the cosmological constant. To summarize, the non-supersymmetric compactifications of M-theory with a vanishing three-dimensional cosmological constant found in [16] are stable to leading order as they minimize the supergravity potential. The eq. (2.13) is the determining equation for the moduli fields. Generically it should be possible to stabilize all of the moduli fields appearing in these compactifications but for the radial modulus as the superpotential is independent of this modulus. This is in contrast to the type-IIB theory considered in [5], where only a superpotential for the complex structure moduli is generated and none of the Kähler moduli fields can be stabilized.

Once supersymmetry is broken by the flux (2.12) the gravitino becomes massive. To see this we notice that the relevant terms of the action of eleven-dimensional supergravity [25]. are

$$L = -\frac{\sqrt{2}\kappa}{384} e \left(\bar{\Psi}_M \Gamma^{MNPQRS} \Psi_S + 12 \bar{\Psi}^N \Gamma^{PQ} \Psi^R \right) F_{NPQR}\,. \tag{2.14}$$

In order to compactify this interaction to three dimensions we decompose the eleven-dimensional gamma matrices as in [17] while the eleven-dimensional gravitino is decomposed as

$$\Psi_\mu = \psi_\mu \otimes \epsilon + {\psi^*}_\mu \otimes \epsilon^* + \cdots , \tag{2.15}$$

where ψ_μ is the three-dimensional gravitino and ϵ is a complex eight-dimensional spinor on the internal manifold with unit norm. In the above formula there are further terms that are necessary in order to eliminate the mixed components of the gravitino mass matrix. Taking this decomposition into account we obtain the following mass terms for the gravitino[7]

$$\bar{\psi^*}_\mu \Gamma^{\mu\nu} \psi_\nu \int_{Y_4} \Omega \wedge F + \text{c.c.} \tag{2.16}$$

for F of type $(0,4)$, while for F non primitive the mass term becomes

$$\bar{\psi}_\mu \Gamma^{\mu\nu} \psi_\nu \int_{Y_4} J \wedge J \wedge F + \text{c.c.}\,. \tag{2.17}$$

Here we have defined $\bar{\psi}_\mu(x) = \psi_\mu^T \Gamma_0$. From the interactions (2.16) and (2.17) we see that the gravitino mass vanishes for supersymmetric compactifications in which $F_{0,4} = F_{4,0} = 0$ and $F \wedge J = 0$ holds, respectively.

3. Higher order corrections to the potential in type-IIB theory

Our next goal is to analyze the effects of higher order terms (like $F^2 R^3$ terms) appearing in the M-theory effective action on the scalar potential. Unfortunately only a few results are known about these terms so that a direct derivation of the corrections to the potential via a Kaluza-Klein reduction is unfeasible at present. The situation for the type-IIB theory is not much better. Some of the relevant terms have been computed in [26]–[29] but a complete calculation of all the leading higher order interactions is still lacking.[8] As mentioned in the introduction, in the type-IIB theory compactified on a Calabi-Yau threefold with fluxes (and external brane sources) a potential of no-scale type is also induced [5] and its structure is similar to the potential discussed in the previous section. Correspondingly it was conjectured that α'- and string loop-corrections generate a potential for the radial modulus so that the no-scale structure is lost [5]. Fortunately, in these compactifications there is a different way to obtain the α'-corrections to the potential which uses mirror symmetry and the c-map of [31].

Let us first recall that the metric for the Kähler deformations of the threefold receives perturbative α'^3-corrections from higher derivative terms appearing in the ten-dimensional type-II effective action (at tree-level the relevant terms coincide for the type-IIA and IIB theory). This has been shown in [32] using the results of [33]–[36]. The relevant terms

[7] Strictly speaking the correct mass terms are obtained after the Weyl rescaling to the Einstein frame.

[8] For a review with a complete list of references see [30]. The effect of such higher order interactions on the dual confining gauge theory has recently been analyzed in [28].

in the type-II effective action responsible for the correction of the Kähler moduli space metric are

$$S = -\frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-g^{(10)}}e^{-2\phi}\left(R + 4(\partial\phi)^2 + \alpha'^3 c_1 J_0\right), \tag{3.1}$$

where $c_1 = \zeta(3)/3 \cdot 2^{11}$. The higher order interaction is defined as

$$J_0 = t^{M_1N_1\dots M_4N_4} t_{M_1'N_1'\dots M_4'N_4'} R^{M_1'N_1'}{}_{M_1N_1}\dots R^{M_4'N_4'}{}_{M_4N_4} + \frac{1}{4}E_8\,, \tag{3.2}$$

where capital letters indicate ten-dimensional indices and ϕ is the ten-dimensional type-II dilaton. The tensor t is defined as in [28] and we are using these conventions in the following. E_8 is a ten-dimensional generalization of the eight-dimensional Euler density given by

$$E_8 = \frac{1}{2}\epsilon^{ABM_1N_1\dots M_4N_4}\epsilon_{ABM_1'N_1'\dots M_4'N_4'} R^{M_1'N_1'}{}_{M_1N_1}\dots R^{M_4'N_4'}{}_{M_4N_4}\,. \tag{3.3}$$

In appendix A we show that we also need a term

$$\sim \frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-g^{(10)}}e^{-2\phi}\alpha'^3\left(\nabla^2\phi\right)Q\,, \tag{3.4}$$

where Q is a generalization of the six-dimensional Euler integrand, $\int_{Y_3} d^6x\sqrt{g}Q = \chi$, which is explicitly defined in (A.5). This term does not modify the equations of motion to order $\mathcal{O}(\alpha'^3)$ but is necessary in order to derive the correct four-dimensional low energy effective action to that order. This is shown in detail in appendix A.

After compactification on a Calabi-Yau threefold the interactions (3.1) together with (3.4) give the perturbative correction to the metric on the moduli space of the Kähler deformations computed in [36]. In particular, it has been shown in [36] how this correction modifies the prepotential of the Kähler moduli space in Calabi-Yau compactifications of the type-II theories. In the type-IIB case the Kähler deformations reside in $\mathcal{N} = 2$ hypermultiplets. However, we are actually interested in orientifold and F-theory compactifications, so that part of the fields appearing in these compactifications have to be projected out.[9] More precisely in the type-IIB hypermultiplet moduli space we need to perform a truncation to an $\mathcal{N} = 1$ subsector by projecting out the moduli that arise from the ten-dimensional antisymmetric two-forms.

We first need to know the corrections to the metric on the hypermultiplet moduli space. It has been shown in [31] that this metric is entirely expressible in terms of the prepotential, via the c-map. Thus all the perturbative corrections to the hypermultiplet moduli space are captured by those to the prepotential calculated in [36]. However, the hypermultiplet moduli space has been parametrized in [31] in variables whose relation to the ten-dimensional type-IIB fields, on which the truncation acts naturally, are not obvious. In fact the transformation relating the two field bases is rather involved and has been established in [37].[10] Thus we first have to translate the perturbative corrections to the hypermultiplet action into the appropriate type-IIB variables.

[9]Strictly speaking our formulas only apply to the Calabi-Yau orientifold case discussed in [5].
[10]For the special case $h^{1,1} = 1$ see also [38].

Let us start from the action of the hypermultiplets in type-IIB compactifications on the Calabi-Yau threefold Y_3 given in [31][11]

$$
\begin{aligned}
-\sqrt{-g}^{-1} L \; &= \frac{1}{2} R - G_{a\bar{b}}(z,\bar{z}) \partial^\mu z^a \partial_\mu \bar{z}^{\bar{b}} - (\partial^\mu \phi_B)^2 - \\
&- \frac{1}{4} e^{4\phi_B} \left(\partial_\mu \tilde{\phi} + \zeta^i \partial_\mu \tilde{\zeta}_i - \tilde{\zeta}_i \partial_\mu \zeta^i \right)^2 + \frac{1}{2} e^{2\phi_B} \partial_\mu \zeta^i R_{ij}(z,\bar{z}) \partial^\mu \zeta^j + \\
&+ \frac{1}{2} e^{2\phi_B} \left(I_{ik}(z,\bar{z}) \partial^\mu \zeta^k + \partial^\mu \tilde{\zeta}_i \right) R^{-1ij}(z,\bar{z}) \left(I_{jl}(z,\bar{z}) \partial_\mu \zeta^l + \partial_\mu \tilde{\zeta}_j \right), \qquad (3.5)
\end{aligned}
$$

where the z^a, $a = 1, \ldots, h^{1,1}(Y_3)$, are the Kähler deformations of Y_3. They are related to the projective coordinates X^i, $i = 0, \ldots, h^{1,1}(Y_3)$, according to

$$
z^i = \frac{X^i}{X^0}, \qquad (\text{i.e. } z^0 = 1). \qquad (3.6)
$$

The ζ^i, $\tilde{\zeta}^i$ and $\tilde{\phi}$ in (3.5) are related to the scalars arising in the R-R sector and the scalar dual to the NS-NS antisymmetric tensor in a complicated way [37], whereas ϕ_B is the four-dimensional type-IIB dilaton. All couplings are determined in terms of a holomorphic prepotential $F(X)$ via

$$
\begin{aligned}
R_{ij} &= \text{Re}\, \mathcal{N}_{ij}, \qquad I_{ij} = \text{Im}\, , \mathcal{N}_{ij}, \\
\mathcal{N}_{ij} &= \frac{1}{4} \bar{F}_{ij} - \frac{(Nz)_i (Nz)_j}{(zNz)}, \\
F_{ij} &= \frac{\partial^2 F}{\partial X^i \partial X^j}, \qquad N_{ij} = \frac{1}{4}(F_{ij} + \bar{F}_{ij}), \\
(Nz)_i &= N_{ij} z^j, \qquad (zNz) = z^i N_{ij} z^j, \qquad (3.7)
\end{aligned}
$$

where we use the conventions and notation of [37]. The metric $G_{a\bar{b}}$ is Kähler with a Kähler potential

$$
K = -\ln[X^i \bar{F}_i(\bar{X}) + \bar{X}^i F_i(X)], \qquad (3.8)
$$

that is also expressed in terms of F. The components of the metric then take the form

$$
G_{a\bar{b}} = \frac{\partial^2 K}{\partial z^a \partial \bar{z}^b} = -\frac{1}{zN\bar{z}} \left(N_{ab} - \frac{(N\bar{z})_a (Nz)_b}{(zN\bar{z})} \right). \qquad (3.9)
$$

The prepotential for the Kähler deformations receives perturbative and non-perturbative corrections on the worldsheet. These have been successfully computed [36] using mirror symmetry and the perturbative correction has been identified with the α'-corrections determined in [33, 34, 40, 41]. The perturbative prepotential determined in this way reads

$$
F(X) = \frac{i}{3!} \kappa_{abc} \frac{X^a X^b X^c}{X^0} + (X^0)^2 \xi. \qquad (3.10)
$$

[11] We have adjusted the formula of [31] to our conventions, i.e. the +++ conventions in the language of [39].

The cubic term is the classical contribution with κ_{abc} being the classical intersection numbers of Y_3. The constant term is proportional to the Euler number χ of Y_3 and in appendix A we derive its precise value

$$\xi = -\frac{\chi}{2}\zeta(3)\,. \tag{3.11}$$

It describes the (worldsheet) perturbative quantum corrections. Inserting (3.10) into (3.8) leads to the corrected Kähler potential of the Kähler class deformations z^a

$$K = -\ln\left[-\frac{i}{6}\kappa_{abc}(z^a-\bar{z}^a)(z^b-\bar{z}^b)(z^c-\bar{z}^c)+4\xi\right]\,. \tag{3.12}$$

Finally we need to display the hypermultiplet action (3.5) in terms of the type-IIB field variables using the explicit map given in [37].[12] However, here we are interested in truncating the spectrum to an $\mathcal{N}=1$ subsector in which the fields coming from the type-IIB antisymmetric two-forms are projected out. In this subsector the map from [37] takes the form

$$\begin{aligned}\zeta^0 &= \sqrt{2}l\,,\\ \tilde{\zeta}_a &= -\frac{\sqrt{2}}{4}g_a\,,\\ \phi_B &= \phi_4\,,\\ \mathrm{Im}(z^a) &= -v^a\,,\end{aligned} \tag{3.13}$$

with all other fields being projected out. In this formula we have used the notation of [37] in which l is the R-R scalar of type-IIB, ϕ_4 the four-dimensional type-IIB dilaton, g_a are the scalars dual to the antisymmetric tensors coming from expanding the four-form into a basis of (1,1)-forms and v^a are the Kähler class moduli of Y_3. These fields describe a set of $2(h^{1,1}+1)$ real coordinates. Since the projection (3.13) breaks $\mathcal{N}=2$ supersymmetry to $\mathcal{N}=1$, the quaternionic geometry of the hypermultiplet moduli space must reduce to a Kähler geometry.[13] Hence our next task is to display appropriate complex coordinates in which the truncated hypermultiplet metric is manifestly Kähler. To lowest order in α' and at the string tree-level this has already been done in [5]. The relevant field variables for the case of only one Kähler modulus were found to be $\tau = l + ie^{-\phi_0}$ with ϕ_0 being the ten-dimensional type-IIB dilaton to leading order and $\rho = \frac{1}{3}g + ie^{4u}$ with e^{6u} being the volume of Y_3 in the Einstein frame. Our goal now is to find a definition of τ and ρ which takes into account the higher order correction appearing in the prepotential (3.10) and which is valid for the case of more than one Kähler modulus. In order to do so we need to express the four-dimensional dilaton appearing in (3.13) in terms of the ten-dimensional dilaton. From the work [32, 33, 44, 45, 46] and it is known that the equation of motion for the

[12]The map in [37] has been interpreted there as the mirror map relating the type-IIA and type-IIB hypermultiplet sectors. It can, however, equally well been understood just inside the type-IIB theory, relating two coordinate bases of the hypermultiplet sector, the one used in [31] and the one which naturally arises in a Kaluza-Klein reduction.

[13]The general situation of truncating an $\mathcal{N}=2$ theory to $\mathcal{N}=1$ is discussed in [42, 43]. We have checked that the projection (3.13) is consistent with the formalism described in [42].

ten-dimensional dilaton gets modified in the presence of the higher derivative term (3.1) such that a constant dilaton is not a solution anymore. Rather, the solution becomes

$$\phi = \phi_0 + \frac{\zeta(3)}{16}\, Q\,, \tag{3.14}$$

where $\phi_0 = \phi_0(x)$ is the uncorrected, constant dilaton and Q is defined in (A.5). The value of the constant appearing in the correction term of (3.14) is determined in appendix A. Here and in the following we set $2\pi\alpha' = 1$ which implies [47]

$$2{\kappa_{10}}^2 = (2\pi)^7(\alpha')^4 = (2\pi)^3\,. \tag{3.15}$$

The α'-dependence of our formulas can be easily restored at the end by dimensional analysis. In order to find the right definition of the four-dimensional dilaton in terms of the ten-dimensional dilaton to order $\mathcal{O}(\alpha'^3)$ one has to compactify the action (3.1) together with (3.4) in the background (3.14) and determine the function in front of the curvature scalar in four dimensions. This is done in appendix A and leads to the following definition of the four-dimensional dilaton to order $\mathcal{O}(\alpha'^3)$

$$e^{-2\phi_4} = e^{-2\phi_0}\left(\mathcal{V} + \frac{1}{2}\xi\right) \tag{3.16}$$

in terms of the Calabi-Yau volume $\mathcal{V}$ and the higher order correction ξ. Here and in the following we are using the notation:

$$\begin{aligned} \mathcal{V} &= \frac{1}{6}\kappa_{abc}v^a v^b v^c\,, \\ \mathcal{V}_a &= \frac{1}{6}\kappa_{abc}v^b v^c\,, \\ \mathcal{V}_{ab} &= \frac{1}{6}\kappa_{abc}v^c\,. \end{aligned} \tag{3.17}$$

Transforming (3.16) into the Einstein frame gives

$$e^{-2\phi_4} = e^{-1/2\phi_0}\left(\hat{\mathcal{V}} + \frac{1}{2}\hat{\xi}\right), \tag{3.18}$$

where we have defined $\hat{\xi} = \xi e^{-3/2\phi_0}$ and used

$$v^a = \hat{v}^a e^{\phi_0/2} \tag{3.19}$$

in order to relate the Kähler moduli in the string frame to those in the Einstein frame.[14] These are the relevant variables in order to make contact with [5]. Without the higher derivative correction $\mathrm{Im}(\tau)$ was defined as the four-dimensional fluctuations around the constant dilaton background ϕ_0. We keep this definition of τ in the following and continue to define $\tau = l + ie^{-\phi_0}$ as in [5].

Inserting (3.13), (3.18) and (3.19) into (3.5) we arrive at the following lagrangian for the $\mathcal{N} = 1$ truncation

$$-\sqrt{-g}^{\,-1}L = \frac{1}{2}R + \frac{2e^{\phi_0/2}}{\hat{\xi} + 2\hat{\mathcal{V}}}\left[R_{00}((\partial_\mu l)^2 + e^{-2\phi_0}(\partial_\mu\phi_0)^2) + R^{-1}_{ab}(x_a x_b + y_a y_b)\right], \tag{3.20}$$

[14] Our definition of the ten-dimensional Einstein frame proceeds via a Weyl-rescaling $g^{(S)}_{MN} = e^{1/2\phi_0} g^{(E)}_{MN}$.

where

$$x_a = I_{a0}\partial_\mu l - \frac{1}{4}\partial_\mu g_a \,,$$
$$y_a = -e^{-\phi_0} I_{a0}\partial_\mu \phi_0 - \frac{3}{4}\partial_\mu \hat{\mathcal{V}}_a \,. \tag{3.21}$$

It is straightforward to compute the non-vanishing components of the couplings

$$\begin{aligned} I_{a0} &= \frac{9}{4} e^{\phi_0} \left(\frac{\hat{\xi}\hat{\mathcal{V}}_a}{\hat{\xi} - 4\hat{\mathcal{V}}} \right), \\ R_{00} &= \frac{1}{2} e^{3\phi_0/2} \left(\frac{2\hat{\mathcal{V}}^2 - \hat{\mathcal{V}}\hat{\xi} - \hat{\xi}^2}{\hat{\xi} - 4\hat{\mathcal{V}}} \right), \\ R_{ab} &= \frac{3}{2} e^{\phi_0/2} \left(\hat{\mathcal{V}}_{ab} + 6\frac{\hat{\mathcal{V}}_a\hat{\mathcal{V}}_b}{\hat{\xi} - 4\hat{\mathcal{V}}} \right), \end{aligned} \tag{3.22}$$

while all other components vanish. The sigma model metric displayed in (3.20) is indeed Kähler which becomes manifest in the Kähler coordinates[15]

$$T^a = \frac{1}{3} g^a + i\hat{\mathcal{V}}^a \,,$$
$$\tau = l + i e^{-\phi_0} \,. \tag{3.23}$$

The corrected Kähler potential takes the following form:

$$\begin{aligned} \mathcal{K} &= -\ln[-i(\tau - \bar{\tau})] - 2\ln\left(-i(T^a - \bar{T}^a)\hat{v}_a + \xi \left(\frac{-i(\tau - \bar{\tau})}{2} \right)^{3/2} \right) - \ln\left[-i\int_{Y_3} \Omega \wedge \bar{\Omega} \right] \\ &= \phi_0 - 2\ln\left(\hat{\mathcal{V}} + \frac{1}{2}\xi e^{-3\phi_0/2} \right) - \ln\left[-i\int_{Y_3} \Omega \wedge \bar{\Omega} \right] + \text{const.}\,, \end{aligned} \tag{3.24}$$

where $\hat{v}_a$ in the first line is understood to be a function of $-i(T^a - \bar{T}^a)$ given by the inverse of (3.17). The last term in the above expression is the Kähler potential for the complex structure moduli.[16]

Before we proceed to determine the corrections to the supergravity potential implied by the corrections to the Kähler potential, let us discuss the symmetries of the four-dimensional effective theory. These are important for arguing that all α'-corrections to the potential originate from a correction to the Kähler potential, while the superpotential is not corrected. The real parts of the Kähler coordinates T^a originate from the ten-dimensional R-R four-form and inherit from its gauge invariance a Peccei-Quinn (PQ) shift symmetry, which is not broken by the α'^3-corrections [5]. Also τ has a shift symmetry which is a special case of the $\mathrm{SL}(2,\mathbb{Z})$ symmetry of the type-IIB theory. However, the $\mathrm{SL}(2,\mathbb{Z})$ symmetry naturally acts on the ten-dimensional dilaton $\tau_{10} = l + ie^{-\phi}$ according to

$$\tau_{10} \longrightarrow \tau'_{10} = \frac{a\tau_{10} + b}{c\tau_{10} + d}\,, \qquad a, \ldots, d \in \mathbb{Z}\,, \qquad ad - bc = 1\,, \tag{3.25}$$

[15] The indices are raised with δ^{ab}.

[16] In the case of Calabi-Yau orientifolds the complex structure moduli are restricted to those even under the orientifolding [5].

and therefore has no obvious action on the four-dimensional τ defined in (3.23). The origin of this problem is that the derivation of (3.20) and therefore of the Kähler coordinates (3.23) and the Kähler potential (3.24) implicitly makes use of a background for the ten-dimensional dilaton given by (3.14) and a constant l. However, this configuration is not invariant under a general $\mathrm{SL}(2,\mathbb{Z})$ transformation as only the subgroup of (3.25) with $c = 0$ leaves l constant. This subgroup is still manifest in the four-dimensional effective theory (including the α'^3-corrections to the potential) and in particular contains the shift symmetry of l.

We now compute the form of the corrected supergravity potential. The $\mathcal{N} = 1$ scalar potential reads

$$V = \frac{e^{\mathcal{K}}}{2\kappa_{10}^2}\Big(G^{-1I\bar{J}} D_I W D_{\bar{J}} \bar{W} - 3|W|^2\Big) , \tag{3.26}$$

where I, J label the scalar fields of the theory. The Kähler covariant derivatives are given by $D_I W = (\partial_I + (\partial_I \mathcal{K}))W$. In [5] it was shown that to leading order, i.e. for $\xi = 0$, and $h_{11} = 1$ the potential derived via a Kaluza-Klein reduction is indeed of the form (3.26) with the Kähler potential (3.24) evaluated at $\xi = 0$. The superpotential for the complex structure moduli fields is

$$W = \int_{Y_3} G_{(3)} \wedge \Omega . \tag{3.27}$$

This superpotential has first been derived in [11] in the context of $\mathcal{N} = 2$ compactifications of the type-IIB theory with fluxes. Here Ω is the $(3,0)$-form of the Calabi-Yau manifold Y_3 and $G_{(3)}$ is the complex three-form

$$G_{(3)} = F_{(3)} - \tau H_{(3)} , \tag{3.28}$$

which transforms as

$$G_{(3)} \longrightarrow \frac{G_{(3)}}{c\tau + d} \tag{3.29}$$

under an $\mathrm{SL}(2,\mathbb{Z})$-transformation (3.25).[17] At the same time the Kähler potential (3.24) at $\xi = 0$ transforms as

$$\mathcal{K}^{(0)} \longrightarrow \mathcal{K}^{(0)} + \ln(c\tau + d) + \ln(c\bar{\tau} + d) . \tag{3.30}$$

Thus to lowest order in α' an $\mathrm{SL}(2,\mathbb{Z})$-transformation (3.25) acts as a Kähler transformation in the low energy effective action, which is therefore left invariant. Thus the fact that the shift symmetry of τ is actually a special case of an $\mathrm{SL}(2,\mathbb{Z})$-transformation is the reason why the dilaton is allowed to appear in the superpotential (3.27), albeit only in the combination (3.28).

Generalizing to $\xi \neq 0$ we note that (3.27) is still the relevant superpotential in this case, such that all the corrections to the potential come from corrections to the Kähler potential (3.24). This is because the superpotential does not receive any α'-corrections, as has been argued in [5] using the argument of [48] that the PQ symmetry of the Kähler moduli T^a forbids their appearance in the superpotential. However, as in the lowest order

[17]Note that to lowest order in α' the dilaton τ coincides with τ_{10}.

case the appearance of the dilaton in the combination (3.28) is possible because its unbroken shift-symmetry is a special case of an $\mathrm{SL}(2,\mathbb{Z})$-transformation. Thus (3.27) should still be the relevant superpotential in the case at hand.

It was shown in [5] that to leading order in α' the W^2-term in (3.26) cancels out leaving a non-negative potential of no-scale type. At the minimum of this potential (i.e. at $V=0$) the type-IIB complex structure moduli can be fixed but all the Kähler moduli (including the radial modulus) remain undetermined [5]. In order to compute the corrections to the potential we can use the superspotential (3.27) in (3.26) and the corrected Kähler potential (3.24). This leads to the following form of the potential

$$V = \frac{e^{\mathcal{K}}}{2\kappa_{10}^2}\Bigg[(G^{-1})^{\alpha\bar\beta} D_\alpha W D_{\bar\beta}\bar W + (G^{-1})^{\tau\bar\tau} D_\tau W D_{\bar\tau}\bar W - \\ - 9\frac{\hat\xi\hat{\mathcal{V}}e^{-\phi_0}}{(\hat\xi-\hat{\mathcal{V}})(\hat\xi+2\hat{\mathcal{V}})}(W D_{\bar\tau}\bar W + \bar W D_\tau W) - 3\hat\xi\frac{\hat\xi^2+7\hat\xi\hat{\mathcal{V}}+\hat{\mathcal{V}}^2}{(\hat\xi-\hat{\mathcal{V}})(\hat\xi+2\hat{\mathcal{V}})^2}|W|^2\Bigg] . \quad (3.31)$$

Here we have restored the dependence on Newton's constant. Clearly the modified potential does not exhibit the no-scale structure of the tree-level potential any more due to its non-trivial dependence on the radial mode ρ and in particular the $|W|^2$-term does not drop out. This means that breaking supersymmetry via[18] $W \neq 0$ does not lead to a vanishing vacuum energy any more. We expect that a similar result is valid for the non-supersymmetric fluxes in M-theory [16], which also lead to a vanishing cosmological constant at leading order. This is suggested by the relationship between type-IIB compactifications with three-form flux and M-theory compactifications with four-form flux [18, 24].

Further corrections to the Kähler potential may arise due to the presence of the orientifold O3-planes and D3-branes. Certainly D3-branes would give rise to additional moduli that we did not take into account. It is however possible to cancel the O3-plane charge totally by the flux, so that in this case no D3-branes have to be included [6]. The only effects of the localized sources that we make use of are that they break supersymmetry by one half and that they modify the Bianchi identity for the five-form field strength. In this way they guarantee the possibility to turn on non-trivial fluxes. However, introducing the localized sources also leads to a non-trivial warp factor. It has been argued in [5, 7] that its effect is subleading in the large volume limit. But also the α'^3 corrections to the potential are suppressed in this limit, cf. (3.32). It would be interesting to get a better understanding of the effects of a non-trivial warp factor on the potential.

As the type-IIB theory also contains correction terms of higher order in α' than $\mathcal{O}(\alpha'^3)$ we can probably trust our result (3.31) only to order $\mathcal{O}(\alpha'^3)$. Let us therefore explicitly exhibit the correction terms up to that order compared to the tree-level potential. Expanding (3.31) they are found to be

$$\delta V = -\frac{\hat\xi}{\hat{\mathcal{V}}}V_{\mathrm{tree}} + \frac{3}{8}\frac{e^{\mathcal{K}^{(0)}}}{\kappa_{10}^2}\frac{\hat\xi}{\hat{\mathcal{V}}}\mid W + (\tau-\bar\tau)\tilde D_\tau W\mid^2 , \quad (3.32)$$

[18] Note that the supersymmetry conditions arising from $D_I W = 0$ are the same as to lowest order. In particular they still demand $W = 0$.

where $\mathcal{K}^{(0)}$ is the tree-level Kähler potential

$$\mathcal{K}^{(0)} = -\ln[-i(\tau-\bar{\tau})] - 2\ln[\hat{\mathcal{V}}] - \ln\left[-i\int_{Y_3}\Omega\wedge\bar{\Omega}\right] \tag{3.33}$$

and [5]

$$\tilde{D}_\tau W = \left(\partial_\tau + \mathcal{K}^{(0)}_\tau\right) W = \frac{1}{\bar{\tau}-\tau}\int_{Y_3}\bar{G}_{(3)}\wedge\Omega\,. \tag{3.34}$$

The first correction term of (3.32) can be entirely understood from a Weyl-rescaling to the Einstein frame after the compactification. More explicitly, reducing the ten-dimensional action in the Einstein frame leads to a non-canonically normalized Einstein-Hilbert term in four dimensions (compare appendix A)

$$\begin{aligned} &-\frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-g}\left(1-\frac{\zeta(3)}{4}Qe^{-3/2\phi_0}\right)R^{(10)} \longrightarrow \\ &\longrightarrow -\frac{1}{2\kappa_4^2}\int d^4x\sqrt{-g}\left(\hat{\mathcal{V}}+\frac{\hat{\xi}}{2}\right)R^{(4)}\,. \end{aligned} \tag{3.35}$$

Performing the Weyl-rescaling leads to the first correction term to the potential.

The second term of (3.32) is more interesting. Using (3.34) and (3.27) it is straightforward to verify that it can be written as

$$-\frac{3}{8}(\tau-\bar{\tau})^2\frac{e^{\mathcal{K}^{(0)}}}{\kappa_{10}^2}\frac{\hat{\xi}}{\hat{\mathcal{V}}}\int_{Y_3}H_{(3)}\wedge\Omega\int_{Y_3}H_{(3)}\wedge\bar{\Omega}\,. \tag{3.36}$$

This can be partly understood as follows. In the ten-dimensional type-IIB action in the string frame there is a term

$$\frac{1}{4\kappa_{10}^2}\int d^{10}x\sqrt{-g}e^{-2\phi}\frac{1}{3!}H_{MNP}H^{MNP}\,. \tag{3.37}$$

Using (3.14) and performing the Weyl-rescaling to the Einstein frame leads to a term

$$\frac{1}{4\kappa_{10}^2}\int d^{10}x\sqrt{-g}e^{-5/2\phi_0}\left(-\frac{\zeta(3)}{8}\right)Q\frac{1}{3!}H_{MNP}H^{MNP}\,. \tag{3.38}$$

Reducing it on Y_3 leads to a term of the form (3.36). However, it does not reproduce the right factor and in addition from the reduction of (3.38) one would also expect further terms involving the (2,1)-forms instead of the (3,0)-form Ω, cf. [5, the appendix]. It is now natural to speculate that the higher derivative corrections to the ten-dimensional type-IIB action provide the missing terms in order to give the result (3.36) in the reduction. It would be interesting to pursue this further and try to see if one might be able to put constraints on possible higher derivative corrections involving two powers of G and three powers R by demanding that they reproduce (3.36). For example one might find that both $\sim (G^2R^3 + \text{c.c.})$- and $\sim G\bar{G}R^3$-terms have to be present in ten dimensions.[19]

[19] This seems to be indicated by the fact that the correction terms in (3.36) only depend on $H_{(3)}$ and not on $F_{(3)}$.

However, a term $\sim G\bar{G}R^3$ is not to be expected from an analysis using the linearized type-IIB supersymmetry.[20] On the other hand, doubts have recently arisen that the superspace approach to the linearised type-IIB theory based on the scalar superfield of [49] is capable of capturing all the results of a string amplitude calculation performed in [29]. Moreover, a $G\bar{G}R^3$-term might be related to an F^2R^3-term in M-theory. An argument for its presence has been put forward in [50] based on the on-shell supergravity superfield of eleven-dimensional supergravity [51]. Furthermore, it seems to be necessary in order to explain the corrections to the universal hypermultiplet in M-theory [52]. Finally, the corrections to the Kähler potential (3.24) also lead to modifications of the kinetic terms for g_a, the moduli stemming from the four-form. Their occurrence indicates that there should be a term $\sim F_{(5)}^2 R^3$ in the ten-dimensional action.

We had seen in section 2 that the non-supersymmetric solutions that we have derived are stable to leading order as they originate from the minimum of a positive potential. Here we see that for compactifications on manifolds with $\chi = 0$ the solutions are still stable as the corrections to the supergravity potential are vanishing in this case. Furthermore, from formula (3.32) we see that the radial modulus is still not stabilized to order $\mathcal{O}(\alpha'^3)$, as we observe a runaway behaviour for it. We expect that additional corrections in α' indeed lead to a stabilization of the radial modulus because they will come with a different power of the volume. If this was the case a cosmological constant could be generated at higher order in α'. It is tempting to speculate that the sign of the cosmological constant might be positive and that we could find de Sitter space as a space-time background. It would certainly be wonderful if we could predict the phenomenological correct relation between the supersymmetry breaking scale and the cosmological constant [53] in this way.

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A. Fixing the constants in (3.10), (3.14) and (3.16)

In this appendix we give some details of the derivation of the constants appearing in (3.10) and (3.14) and of the relation (3.16).

[20] M.H. thanks M.B. Green and M. Bianchi for interesting discussions on this point.

The equation of motion for the dilaton stemming from

$$S = -\frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-g^{(10)}} e^{-2\phi} (R + 4(\partial\phi)^2 + (\alpha')^3 c_1 J_0) \,, \tag{A.1}$$

to order $\mathcal{O}(\alpha'^3)$ is

$$R + 4\nabla^2\phi - 4(\nabla\phi)^2 + \alpha'^3 c_1 J_0 = 0 \,. \tag{A.2}$$

We introduce complex coordinates on Y_3,i.e.

$$\xi^a = \frac{1}{\sqrt{2}}(y^{2a-1} + iy^{2a}) \,, \qquad \text{for } a = 1, \ldots, 3 \,. \tag{A.3}$$

J_0 has the property of vanishing on Ricci flat Kähler spaces. Therefore it does not contribute to (A.2).

It has been shown in [40, 41] that up to four loops the metric beta-function for the two-dimensional $\mathcal{N} = 2$ non-linear sigma model is given by[21]

$$\beta_{a\bar{b}} = \frac{1}{2\pi} R_{a\bar{b}} + \frac{1}{8\pi} \zeta(3) \nabla_a \nabla_{\bar{b}} Q \,, \tag{A.4}$$

where we have again used $2\pi\alpha' = 1$. The explicit expression for Q is

$$Q = \frac{1}{12(2\pi)^3} \left(R_{IJ}{}^{KL} R_{KL}{}^{MN} R_{MN}{}^{IJ} - 2 R_I{}^K{}_J{}^L R_K{}^M{}_L{}^N R_M{}^I{}_N{}^J \right) . \tag{A.5}$$

For a six-dimensional manifold it is the Euler density, i.e. $\int_{Y_3} d^6x \sqrt{g} Q = \chi$.

Demanding $\beta_{a\bar{b}} = 0$ and using (A.2) and (A.4) we see that the equation of motion is satisfied to order $\mathcal{O}(\alpha'^3)$ if

$$\phi = \phi_0 + \frac{\zeta(3)}{16} Q \,, \tag{A.6}$$

where ϕ_0 is a constant.

We now turn to the determination of the constant ξ appearing in (3.10). It is independent of the number of Kähler moduli and therefore we consider the case of a single modulus here, i.e. we make the following Ansatz for the metric in the string frame

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu + e^{2u} \tilde{g}_{mn} dy^m dy^n \,. \tag{A.7}$$

We choose the volume of the background manifold measured by $\tilde{g}$ to be $(2\pi\alpha')^3 = 1$, such that $\kappa_4^{-2} = \kappa_{10}^{-2}$. We normalize the single (1,1)-form in such a way that we have $\mathcal{V} = e^{6u} = v^3$, cf. (3.17).

Fixing the constant in (3.10) requires a reduction of (3.1) (augmented by the term (3.4), determining the kinetic term of u and comparing with the one obtained from (3.12), i.e.

$$S = \frac{1}{\kappa_4^2} \int d^4x \sqrt{-g} \Big(-(3 - 6\xi e^{-6u}) \partial_\mu u \partial^\mu u \Big) + \cdots \,. \tag{A.8}$$

In order to derive (A.8) we have used that (3.12) is the Kähler potential for the metric $G_{a\bar{b}}$ of the Kähler moduli for both type-IIA and type-IIB on Y_3 and the z^a stand for the Kähler moduli $z^a = b^a - iv^a$.

[21] Note that we use a convention for the Ricci-tensor that differs by a sign from the one used in [41].

In the reduction of J_0 we make use of the fact that it can be expressed as [45][22]

$$S = -\frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-g^{(10)}}e^{-2\phi}\alpha'^3 c_2\left(12Z - RS + 12R_{MN}S^{MN} + (\text{Ricci})^2\right) + \\ + \cdots\,, \tag{A.9}$$

where we have introduced a new constant $c_2 = \zeta(3)/3\cdot 2^5$ and used the notation of [45], i.e. $Z = Z_{IJ}g^{IJ}$, $S = S_{IJ}g^{IJ}$ with

$$\begin{aligned} Z_{IJ} &= R_{IKLR}R_{JMN}{}^{R}\left(R^{K}{}_{P}{}^{M}{}_{Q}R^{NPLQ} - \frac{1}{2}R^{KN}{}_{PQ}R^{MLPQ}\right),\\ S_{IJ} &= -2R_I{}^{MKL}R_J{}^{P}{}_{K}{}^{Q}R_{LPMQ} + \frac{1}{2}R_I{}^{MKL}R_{JMPQ}R_{KL}{}^{PQ} - \\ &\quad - R_I{}^{K}{}_{J}{}^{L}R_{KMNQ}R_L{}^{MNQ}\,. \end{aligned} \tag{A.10}$$

Furthermore, $S = 12(2\pi)^3\,Q + (\text{Ricci})$, where Q has been defined in (A.5).

In order to evaluate (A.9) for the metric (A.7) we need the non-vanishing components of the Riemann tensor. Using the conventions

$$\begin{aligned} R^{M}{}_{NPQ} &= \partial_P\Gamma^M_{QN} - \partial_Q\Gamma^M_{PN} + \Gamma^R_{QN}\Gamma^M_{PR} - \Gamma^R_{PN}\Gamma^M_{QR}\,,\\ \Gamma^M_{NP} &= \frac{1}{2}g^{MQ}\Big(\partial_N g_{PQ} + \partial_P g_{QN} - \partial_Q g_{NP}\Big)\,, \end{aligned} \tag{A.11}$$

we find

$$\begin{aligned} R^{m}{}_{\mu n\nu} &= -\delta^m_n(\partial_\mu u\partial_\nu u + \partial_\mu\partial_\nu u)\,,\\ R^{\mu}{}_{m\nu n} &= -g_{mn}(\partial_\nu u\partial^\mu u + \partial_\nu\partial^\mu u)\,,\\ R^{k}{}_{mnp} &= \tilde{R}^k{}_{mnp} + \partial_\mu u\partial^\mu u(\delta^k_p g_{mn} - \delta^k_n g_{pm}) \end{aligned} \tag{A.12}$$

with all other components, not related to these by symmetry, vanishing. Thus the non-trivial components of the Ricci-tensor are[23]

$$\begin{aligned} R_{\mu\nu} &= -6(\partial_\mu u\partial_\nu u + \partial_\mu\partial_\nu u)\,,\\ R_{mn} &= -g_{mn}(6\partial_\mu u\partial^\mu u + \partial_\mu\partial^\mu u)\,, \end{aligned} \tag{A.13}$$

whereas the Ricci scalar is given by

$$R = -42\partial_\mu u\partial^\mu u - 12\partial_\mu\partial^\mu u\,. \tag{A.14}$$

Now we are in a position to discuss the different sources for contributions to the kinetic term of u stemming from (A.9). From (A.13) and (A.14) it is clear that the terms $\sim RS$ and $\sim R_{mn}S^{mn}$ contribute, whereas the $(\text{Ricci})^2$-terms and the one $\sim R_{\mu\nu}S^{\mu\nu}$ do not. More involved is the discussion of the Z-term. From its definition in (A.10) we see that the

[22]See also [54].

[23]Here we skip the contribution of the background metric. According to (A.4) and in complex coordinates it is proportional to $\nabla_a\nabla_{\bar{b}}\tilde{Q}$ and does not contribute in the reduction.

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only contributions come from taking all four Riemann tensors with internal indices only, i.e. those given in the last line of (A.12). To second order in derivatives of u we get

$$Z = e^{-6u}\partial_\mu u\partial^\mu u\Big(12(2\pi)^3\tilde{Q} + \tilde{R}^i{}_{qlr}\left(2\tilde{R}_{ipn}{}^r\tilde{R}^{nplq} - \tilde{R}_{inp}{}^r\tilde{R}^{nlpq} - \tilde{R}_{ipn}{}^r\tilde{R}^{qnlp}\right)\Big)\,, \quad \text{(A.15)}$$

where $\tilde{Q}$ is as in (A.5) but evaluated with the background metric $\tilde{g}_{mn}$. The second cubic polynomial in the Riemann tensor is different from $\tilde{Q}$ for a general manifold. On a Kähler manifold, however, it is possible to show that it is indeed given by $12(2\pi)^3\tilde{Q}$. To see this we again introduce complex coordinates on Y_3. Using the fact that the only non-trivial (independent) component of the Riemann tensor on a Kähler manifold, $\tilde{R}^a{}_{bc\bar{d}}$, has the additional symmetry

$$\tilde{R}^a{}_{bc\bar{d}} = \tilde{R}^a{}_{cb\bar{d}}\,, \quad \text{(A.16)}$$

we derive for Kähler manifolds

$$\begin{aligned} 12(2\pi)^3\tilde{Q} &= 4\left(\tilde{R}_{a\bar{b}}{}^{c\bar{d}}\tilde{R}_{c\bar{d}}{}^{e\bar{f}}\tilde{R}_{e\bar{f}}{}^{a\bar{b}} - \tilde{R}_a{}^c{}_b{}^d\tilde{R}_c{}^e{}_d{}^f\tilde{R}_e{}^a{}_f{}^b\right) \\ &= \tilde{R}^i{}_{qlr}\left(2\tilde{R}_{ipn}{}^r\tilde{R}^{nplq} - \tilde{R}_{inp}{}^r\tilde{R}^{nlpq} - \tilde{R}_{ipn}{}^r\tilde{R}^{qnlp}\right). \end{aligned} \quad \text{(A.17)}$$

Thus on a Kähler manifold (A.15) simplifies and becomes

$$Z = 24(2\pi)^3 e^{-6u}\partial_\mu u\partial^\mu u\tilde{Q}\,. \quad \text{(A.18)}$$

Let us now perform the reduction of (3.1) with the metric Ansatz (A.7) and the dilaton $\phi = \phi_0 + cQ$, where the constant c has been determined in (A.6) to be $c = \zeta(3)2^{-4}$. As we have already mentioned in the main text we include a term

$$-\frac{a}{2\kappa_{10}^2}\int d^{10}x\sqrt{-g^{(10)}}e^{-2\phi}\alpha'^3 c_2(\nabla^2\phi)S\,. \quad \text{(A.19)}$$

The constant a can be determined as follows. It has been argued in [55] that the Ricci-terms in (A.9) actually appear in the combination $R_{MN} + 2\nabla_M\nabla_N\phi$.[24] Inserting this into (A.9) we get the additional terms

$$S = -\frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-g^{(10)}}e^{-2\phi}\alpha'^3 c_2\Big(-2(\nabla^2\phi)S + 24(\nabla_M\nabla_N\phi)S^{MN}\Big) + \cdots\,. \quad \text{(A.20)}$$

Whereas the second one does not contribute in the reduction, the first one is exactly of the form (A.19) with a constant $a = -2$.

Using this we derive to order $\mathcal{O}(\alpha'^3)$

$$\begin{aligned} S = -\frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-g}e^{-2\phi_0}\Big[&(1-2cQ)\Big(R^{(4)} - 42\partial_\mu u\partial^\mu u - 12\partial_\mu\partial^\mu u + 4\partial_\mu\phi_0\partial^\mu\phi_0\Big) - \\ &- 48cQ\partial_\mu\phi_0\partial^\mu u - 48c_2Q\partial_\mu\phi_0\partial^\mu\phi_0 + \\ &+ 12c_2Q(-R^{(4)} - 6\partial_\mu u\partial^\mu u)\Big]\,. \end{aligned} \quad \text{(A.21)}$$

[24]Note that our definition of the dilaton differs from [55] by a factor of -2 but also the definition of our Ricci-tensor is different by a factor of -1.

Integrating over the internal coordinates and using $6c_2 = \frac{1}{16}\zeta(3) = c$ we arrive at

$$S = -\frac{1}{2\kappa_4^2}\int d^4x\sqrt{-g}e^{-2\phi_0}\Big[(e^{6u}-4c\chi)R^{(4)} + (e^{6u}-2c\chi)(-42\partial_\mu u\partial^\mu u - 12\partial_\mu\partial^\mu u) + \\ + \left(e^{6u}-4c\chi\right)4\partial_\mu\phi_0\partial^\mu\phi_0 - \\ - 48c\chi\partial_\mu\phi_0\partial^\mu u - 12c\chi\partial_\mu u\partial^\mu u\Big]\,. \tag{A.22}$$

From the prefactor of $R^{(4)}$ we can read off the four-dimensional dilaton

$$e^{-2\phi_4} = e^{-2\phi_0}\left(e^{6u}-4c\chi\right). \tag{A.23}$$

Performing a partial integration in the last term of the first row of (refsredfour) it is straightforward to verify that (A.22) can be expressed as[25]

$$S = -\frac{1}{\kappa_4^2}\int d^4x\sqrt{-g}e^{-2\phi_4}\left(\frac{1}{2}R^{(4)} + 2\partial_\mu\phi_4\partial^\mu\phi_4 - [3+48c\chi e^{-6u}]\partial_\mu u\partial^\mu u\right). \tag{A.24}$$

A Weyl-rescaling to the four-dimensional Einstein frame does not alter the coefficient of the kinetic term for u. We can therefore compare it with (A.8) and find

$$\xi = -8c\chi = -\frac{\zeta(3)\chi}{2}\,. \tag{A.25}$$

Inserting this into (A.23) leads to (3.16).

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